Introduction to the Geometry for Teachers course Student Learning Outcomes

What should preservice teachers learn in a college geometry course, and how should they learn it? There has been little nationwide consensus on how best to answer this two-fold question, despite the importance of high school geometry in college preparatory curricula. The geometry for teachers (GeT hereafter) courses taught at colleges and universities across the United States demonstrate a wide variety of content focus, reflecting institutional demands, curricular requirements, and instructor interest (Grover & Connor, 2000; Venema et al., 2015).

In 2017, the University of Michigan GRIP[1] lab, under the direction of Dr. Patricio Herbst and Dr. Amanda Brown, received a National Science Foundation grant for the project "GeT Support: An online professional learning community to support the geometry course for teachers." The grant grew out of Herbst and Brown's commitment to improving preservice teacher (PST) preparation, especially in the content area of geometry. They recruited GeT course faculty from across the country with the intent of creating a community of instructors sharing that commitment. The *Get: a Pencil* Community first convened in Ann Arbor in June 2018, and has continued to have a robust online presence since then. The ongoing focus of *GeT: a Pencil* is to work collectively towards the goal of improving the capacity of secondary geometry teaching.

Since the conference in 2018, the community has continued to meet online to discuss issues relevant to the GeT course. One issue that surfaced early in these discussions was the lack of a clear shared understanding of what should be in the GeT course. A subgroup of the community, the Teaching GeT working group, was formed to try to address this issue, and it is this subgroup that has been responsible for putting together this document. Through our initial conversations, we (the working group) realized that in order to better prepare preservice teachers to teach geometry, we needed a common set of content objectives for the GeT course. Everyone in the Teaching GeT working group had an opportunity to contribute their own list of what content they felt should be included in a GeT course. We spent months meeting online, discussing all suggestions, respectfully disagreeing on items, and narrowing the list down to ten essential student learning objectives (SLOs). Here, *essential* means the identification of content knowledge that all prospective secondary geometry teachers should have the opportunity to learn. For each of these SLOs, there is a brief content description, a paragraph with more details, and a longer narrative that describes the SLO in detail, along with suggestions of specific content to include when covering the SLO.

1. Proofs
2. Critique Reasoning
3. Secondary Geometry Understanding
4. Axiomatic Systems
5. Definitions
6. Technologies
7. Euclid
8. Constructions

[1] Grasping the Rationality of Instructional Practices

9. Non-Euclidean Geometries
10. Transformations

One of the tensions that we faced during the creation of the SLOs was whether or not we should prescribe specific teaching practices for a GeT course. On the one hand, we want to emphasize recommendations for college mathematics instructional practices, such as those espoused by the Mathematical Association of America (Abell et al., 2017), that are particularly relevant to geometry teaching. GeT courses are great places for active and inquiry-based styles of learning, and there are many accessible kinds of problems to be solved. On the other hand, we want to highlight opportunities for GeT instructors to facilitate students' learning of secondary teaching practices, such as those recommended by the NCTM (2020) [See SLO 3]. Because the audience of the GeT course and the course itself currently varies so much between institutions, we want these SLOs to be flexible. One of these flexibilities is that they may be used in a single course or spread throughout various courses in a program. Therefore, we support a pedagogical frame whereby faculty using the SLOs have the academic freedom to make informed decisions regarding the teaching methods used in their programs and courses. Thus, the narratives of the SLOs include references to pedagogical resources and content to make faculty teaching the GeT course aware of these and where to find them.

The group has identified some best practices for the GeT course that we believe should apply to all courses. Some of these practices are specifically outlined in the SLOs. For instance, geometry is a traditional setting for teaching proof, but it is more generally an ideal setting for working on all types of mathematical communication. [See SLO 1 on Proof and SLO 2 on Critiquing Reasoning.] It is important for students to work together to solve problems, and at the same time learn to productively collaborate. Geometry courses are also an ideal setting for allowing students to experience the progression of exploring followed by conjecturing followed by proving. This central mathematical process is one that students may not experience in very many classes in their college career. Geometry courses should introduce Dynamic Geometry Environments [See SLO 6] because they afford students vital opportunities to explore and develop conjectures which can be proven or disproven.

Other times our recommended practices may not link directly to specific SLOs. They instead regard general processes for learning mathematics that we believe students should experience within a GeT course in order to better understand the geometry content. Applying geometry to contexts outside of mathematics and connecting geometry to other mathematical domains are each valuable. Students should have many chances to experience and develop proficiency with the mathematical process skills of problem-solving, oral and written communication of mathematical ideas, and productive collaboration. Some curricula focus on the exploration and conjecturing, while others focus on the proving, but we think that it is most powerful for students to go through this entire process and to try to prove their own conjectures or produce counterexamples. This requires planning and knowledge on the part of the instructor but is well worth it.

References

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